A new deal for vector diagrams

Dear Sir—In a recent article in the February 1964 issue of *Electronics and Power*, p. 47, Mr. Scroggie argued that the present system of vector representation tends to confuse students, because of possible ambiguity with regard to the starting-point of vectors which are in phase and the 'confused mess' arising when two arrowheads meet at a point.

In the first case the fault is possibly one of poor definition of the vector quantities represented, and in the other the fault lies in the possession of a broad-tipped pen or pencil.

While agreeing with Mr. Scroggie that ambiguities do arise when circuits are lettered and two-way voltage arrows and one-way current arrows are included on the diagram, I do not agree that removal of the arrowheads from circuit and vector diagrams clarifies any but the most simple circuit diagrams.

Referring to the circuit and vector diagrams of Figs. 2 and 4 of the article (see my Fig. B), it seems strange that, in the one case, a series of seven possible vector diagrams can be drawn, while for the new system only one possible vector diagram exists. It is agreed that, with the new system of lettering each junction point in the circuit, it becomes almost impossible to make a mistake when drawing the vector diagram, but, with the same degree of logicality, there can be only one method of drawing the vector diagram in Fig. 2.

Mr. Scroggie asks 'can any of them (i.e. the vector diagrams of his Fig. 2) tell us, for example, the potential of the R_1L junction relative to the LR_2 junction at any chosen phase of the e.m.f.?', and his answer is that they cannot. He then goes on to develop his recommended convention, which, it is claimed, is unambiguous, and therefore must indicate the above-required potential difference.

If this is an important consideration, then neither of these two vector diagrams is really satisfactory, since, to determine this potential difference, the potentials of the junctions R_2L, R_1L with respect to the generator–R_2 junction point need to be determined and then subtracted. This potential difference can best be shown by means of a vector $j\omega LI$ radiating from the origin.

Modifying the circuit slightly as in Fig. C, the vector diagrams for the two conventions are as shown.

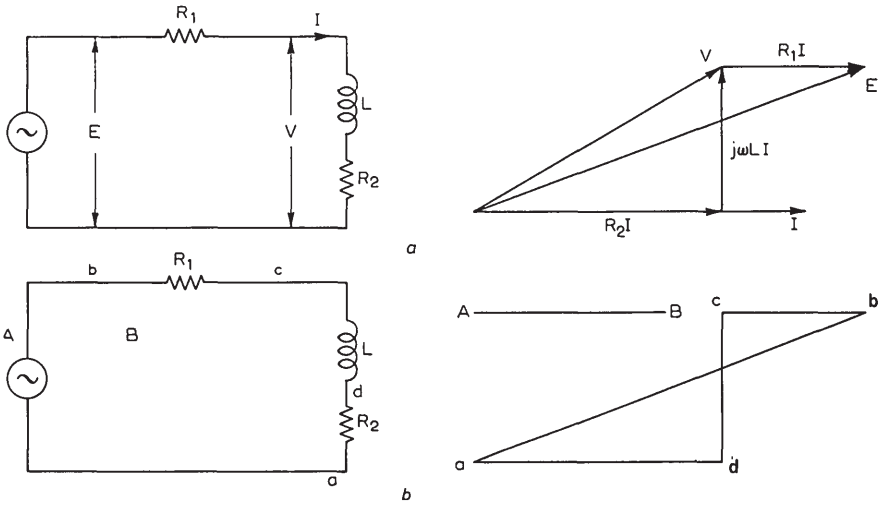
In the usual method of drawing the vector diagram, the four vectors, I, R_2I, R_1I and V, are all in phase. The problem arising in this case is: is the R_1I vector assumed to be drawn from the origin O or from the tip of the R_2I vector? The answer has to be sought from the other condition governing the drawing of vector diagrams, i.e. that the diagram is drawn to represent a vector equation.

By circuit theory, the following equation can be written:

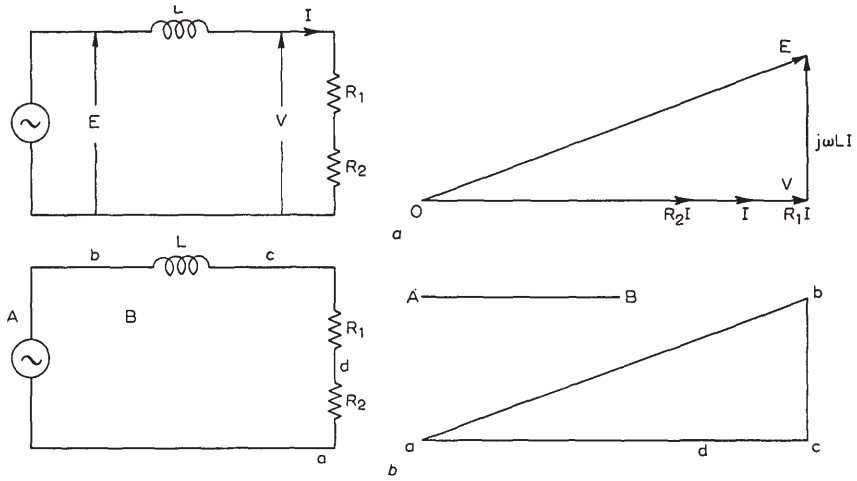
$$V = R_2I + R_1I$$

and taking this equation and the vector diagram into consideration, it becomes evident that, for this vector diagram, the vector of potential difference across R_1 has been drawn from the tip of the R_2I vector, marked with an arrowhead, to the tip of the V vector, also marked with an arrowhead. An arrowhead is thus performing an important function in defining the end of one vector and the commencement of another.

The vector diagram for the new convention now gives rise to an important question, viz. should the point d be marked on the vector diagram? If it is not marked, the point d becomes indeterminate. If it is to be marked without the use of an arrowhead, some other mark needs to be



B *Simple a.c. circuit diagram and its associated vector diagram drawn according to*
a present convention (part of Fig. 2 of M. G. Scroggie's article)
b recommended convention (Fig. 4 of M. G. Scroggie's article)



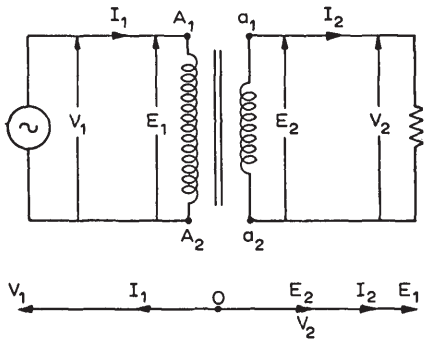
C *Modified a.c. circuit diagram and its associated vector diagram*

a present convention
b recommended convention

adopted. Putting a dot at this point leads to ambiguity as to whether the vector diagram rotates around the dot at point *a* or the dot at point *d*.

The new method does away with the necessity of referring to the voltage equation but introduces its own ambiguities into the vector diagram.

The further query raised by Mr. Scroggie regarding the polarity of the secondary terminals of a transformer, given the polarity of the primary terminals, is completely unanswerable, because he has not adopted the standard conventions for marking the windings of transformers. The ends of transformer windings are both lettered and numbered, and this marking contains a wealth of information. The ends of the primary winding of a single-phase transformer are usually marked A_1 and A_2, and those of the secondary winding, a_1 and a_2. With this convention, it is known that, when terminal A_1 is made positive with respect to A_2, terminal a_1 will be positive with respect to a_2. It is further known that, when a current is



D *Circuit and vector diagram for an ideal transformer*

entering terminal A_1, a current will be leaving terminal a_1.

The circuit and vector diagrams for an ideal 2 : 1-ratio single-phase transformer delivering a secondary current at unity power factor are given in Fig. D.

The vector diagram illustrates the following vector equations:

applied voltage (V_1) + e.m.f. induced in the primary winding (E_1) = 0

$$V_1 + E_1 = 0$$
$$V_1 = -E_1$$

e.m.f. induced in the secondary winding = potential difference across the load

$$E_2 = V_2$$

From the turns ratio of the transformer, further equations can be developed relating the induced e.m.f.s and also the currents.

The primary and secondary e.m.f.s are both induced by the same flux linking with the two windings, and the e.m.f.s will therefore be in phase, so that the e.m.f. relationship can be written vectorially as

$$E_1 = \frac{N_1}{N_2}E_2 = 2E_2$$

For the current relationships, the m.m.f. produced by the secondary current must be neutralised by an m.m.f. produced by the primary winding:

$$N_2I_2 + N_1I_1 = 0$$

Therefore $$I_1 = -\tfrac{1}{2}I_2$$

A convention has to be adopted for power. In this case, positive power is taken as electrical power output from a source, and negative power is electrical power input to a circuit element.

By referring to the vector diagram of Fig. D, the expressions for power can be written as follows:

- supply power is $V_1 \times I_1$; this is positive, since I_1 is in phase with V_1; hence power is taken from the supply
- transformer primary power is $E_1 \times I_1$; this is negative and represents the power input to the primary winding
- transformer secondary power is $E_2 \times I_2$; this is again positive and represents the power output from the secondary winding.

The use of the arrowhead is justified therefore, in that the power equations also indicate whether power is being abstracted from a supply or used in a load.

The new convention, while possibly looking neater, can introduce its own ambiguities, and I feel that, considering a complex system, the removal of arrowheads would result in much more scratching of heads than the present system, provided that a more detailed approach is made to the teaching of vectors than is apparently the case.—Yours faithfully,

C. GOLDING

Royal College of Advanced Technology
Salford, Lancs.
17th March 1964

* * *

Dear Sir—In his letter in the May 1964 issue of *Electronics and Power*, p. 167, Mr. Connelly asks several questions about my article in the February 1964 issue, p. 47. I now answer them in the same order.

So far from being concerned about ascribing a specific polarity to an alternating voltage, I am concerned to liberate vector diagrams from the specific polarity implied by an arrowhead at one end. A vertical line conventionally represents a voltage, which can be either maximum positive or maximum negative, depending on (*a*) which of the two points between which the voltage exists in the circuit is regarded as reference, and (*b*) which end of the vector corresponds to that point. My lettering of circuit points and vector ends, in conjunction with the three natural and generally understood conventions I mentioned, gives a perfectly unambiguous tie-up between vector and circuit diagrams. It is true that half a cycle later the polarity thus indicated will be reversed, but so will that of every other quantity involved, so the relationship between them (which is what we want to know) is unambiguously shown. The use of double-headed arrows in circuit diagrams, on the contrary, makes them fully ambiguous.

The lengths of the vectors in my system, and the angles between them, mean the same things and are determined in the same ways as in other systems.

I agree with Mr. Connelly that the multiplicity of vector diagrams in my Fig. 2 illustrates that vectors are associative and commutative, but does he consider that the permanent existence of so many diverse practices is thereby justified when these well known properties can be taught once for all in one short easy lesson? If not, how else does he justify them? There are many possible ways of indicating to motorists that they are coming to the entrance of a one-way street, but the desirability of having one standard pattern for such a purpose is universally recognised. Similarly, if the kind of electrical conditions in a circuit can be seen at a glance from the shape of the one and only vector diagram logically appropriate to those conditions, much clarity and saving of time and effort are achieved. In the diagrams I advocate, the relative positions of the lines are all unique and meaningful; as Mr. Connelly admits, in other systems they are arbitrary.

Letters to the Editor (continued)

In my Fig. 4, V_{dc} (a vertically upward line) does, as Mr. Connelly suggests, represent the maximum positive voltage across the inductor L. That being so, his query as to whether V_{cb} (a line pointing horizontally to the right) also represents a maximum positive voltage is surely one that would not unduly tax the most elementary student halfway through lesson 1 in a course on *any* system of vector diagrams. However, on the assumption that the question is for information and not to make some debating point obscure to me, I will reply that, in accordance with accepted convention, it represents a voltage which at the instant shown is zero, being 90° lagging with reference to V_{dc}. We can also see at once, for example, that if the point d in the circuit is earthed, the whole of R_1 is momentarily at maximum positive potential. Information of this kind is not obvious at a glance in any of the many conventional alternatives in Fig. 2, which is another reason for their speedy rejection.

Turning now to Mr. Golding's letter, printed above, I find this shows a complete failure to perceive the simplicity of the new system. He says that, in order to determine the potential of c relative to d at any chosen phase of the e.m.f. (which was the example I chose), neither type of diagram is really satisfactory, since the potentials of c and d need to be determined with respect to a and then subtracted. This may be necessary with other systems, but it certainly is not with mine. All one has to do is see how far c lies above d in the vector diagram. It is as simple as that. At the instant illustrated, in which the e.m.f. reckoned with respect to a is 20° from zero towards positive maximum, c is obviously at positive maximum with respect to d.

Mr. Golding's treatment of the problem suggests that he is assuming that point a is earthed. But there is no information to say so. My vector diagram leaves the matter completely open; it is the only kind to be quite unprejudiced. Others make what may be unwarranted and misleading assumptions implied by their disposition of vectors. Yet the moment an earthing point is known, even though it be a point not actually on the circuit, it can at once be indicated on my vector diagram, and the potential of any and every point in the circuit at any phase of any current or voltage can readily be seen by suitably orientating the diagram. This is a facility far beyond any other type of diagram and becomes more valuable the more complicated the circuit. Yet the diagram is manifestly simpler than any other kind. Does Mr. Golding really think it is open to dispute that a total of six letters is neater than ten letters, two subscript figures, one Greek symbol, one extra line and six arrowheads? The atter would be hard to justify unless they gave more information, but in fact they give less.

The rest of Mr. Golding's objections do not arise. In particular, there is absolutely no need to shift components around (quite inadmissible practically in h.f. circuits) or to modify my vector diagram. The only admissible modification is indication of earth point, when known. Nor, of course, does the problem of finding where collinear vectors begin arise, nor does that of finding a sharp-pointed pen or pencil. Anything that can distinguish one letter of the alphabet from another will do. There are no subscripts on the diagrams, either. There is no problem about point d nor need for dots. And, contrary to what Mr. Golding says, there are no ambiguities.

With regard to the transformer (my Fig. 3), which has been commented on by several correspondents, the rotation in which the coils were drawn, being the same for both, supplied all the necessary information; but if the belt-and-braces men like to add the dot notation to make sure, I would not stand in their way. It is, I think, neater and better established than Mr. Golding's.

Lastly, I have received a letter from Dr. D. Midgley of the Bradford Institute of Technology pointing out that even the one weak point—a difficulty (not insuperable) with current notation in 3-phase circuits—which I disclosed on p. 133 of the April issue, was not so weak as I had feared, as there were at least two other and simpler devices for including them in the system.

So far, all criticisms that have been made of the new system have arisen from misunderstandings of it, owing no doubt to my incompetence as an expositor.—Yours faithfully,

M. G. SCROGGIE

18 Bromley Common, Bromley, Kent
23rd April 1964